modelization of the vehicle running behaviour on a track (and Excel sheet notice)

principles

The propulsion force F_p generated at the vehicle motored wheel is used to overcome the mechanical rolling friction on the wheels, the air friction on the body, the gravity force and the inertia forces (linear inertia and rotational inertia) :

$$F_p * ign = a \cdot m \cdot g \cdot \cos\theta + a' \cdot m \cdot \frac{v^2}{\Re} + \frac{1}{2}\rho \cdot S \cdot C_x \cdot v^2 + m \cdot g \cdot \sin\theta + (m + m_i)\frac{dv}{dt} \tag{1}$$

hence
$$\frac{dv}{dt} = \frac{\left[F_p * ign - m \cdot g \cdot (a \cdot \cos\theta + \sin\theta)\right] - (\frac{a' \cdot m}{\Re} + \frac{1}{2}\rho \cdot S \cdot C_x) \cdot v^2}{(m + m_i)} = A - Bv^2 \tag{2}$$

$$dx = v \cdot dt = \frac{v \cdot dv}{A - B \cdot v^2} = \frac{dv^2}{2(A - B \cdot v^2)} = \frac{du}{2B(\frac{A}{B} - u)} \tag{3}$$

where (approximate values for prototypes are given between parenthesis):

a	axial *mechanical friction constant* referred to the weight of the vehicle (0.002 to 0.005).
m	*vehicle mass* with pilot (80 to 110kg)
a'	lateral mechanical friction constant (in a curve) referred to the weight of the vehicle (neglected here).
m_i	equivalent mass for the rotational masses
ρ	density of air (app. 1.2kg/m3, depending on the weather and altitude)
S	frontal area (0.3 to 0.5m2)
C_x	aerodynamic drag coefficient (0.08 to 0.16)
v	instantaneous vehicle speed (for the calculus of the aerodynamic force it would be useful to add to this speed the algebraic value of the axial projection of the wind speed)
θ	angle of the actual road relative to a horizontal reference
R	wheel radius (0.24 to 0.25m)
$\Re$	curve radius (m)
T_m	engine/motor torque (T_m≈0.3 to 10Nm, electric motor to gas/Diesel engine)
$F_p = \frac{T_m \cdot \eta_t}{r \cdot R}$ F_p	propulsive force (function of the engine/motor speed N_m and of the throttle opening α_{gaz})
ign	switch-on symbol (value : 0 if engine is off, 1 if engine is on)
η_t	transmission efficiency (from 0.9 to 0.98)
r	gear ratio (= number of pinion teeth/number of crown teeth, 0.1 as an order of magnitude for one-stage transmission)

Remark: during free-wheeling (coasting) on a steep downhill road, A is usually positive for $-(a \cdot \cos\theta + \sin\theta) > 0$

One considers road sections (between abscissas x_i and x_{i+1} with an initial speed v_i at time t_i) for which *A* and *B* have quasi-constant values ; the solving of equations (2) and (3) under definite integral forms, gives,

* if *A*<0 :
$$t_{i+1} - t_i = \frac{-1}{\sqrt{|A|B}}\left[\tan^{-1}(\sqrt{\frac{B}{|A|}}v_{i+1}) - \tan^{-1}(\sqrt{\frac{B}{|A|}}v_i)\right] \tag{2'}$$

$$x_{i+1} - x_i = \Delta x_i = \frac{-1}{2B}\left[\log(\frac{|A|}{B} + v_{i+1}^2) - \log(\frac{|A|}{B} + v_i^2)\right] = \frac{-1}{2B}\left[\log\left|1 - \frac{B}{A}v_{i+1}^2\right| - \log\left|1 - \frac{B}{A}v_i^2\right|\right] \tag{3'}$$

Remark : letting $\sqrt{\frac{|A|}{B}} = v_0^-$ (constant), equations (2') and (3') become

$$t_{i+1} - t_i = \frac{-1}{B \cdot v_0^-}\left[\tan^{-1}(\frac{v_{i+1}}{v_0^-}) - \tan^{-1}(\frac{v_i}{v_0^-})\right] \tag{2''}$$

$$x_{i+1} - x_i = \Delta x_i = \frac{-1}{2B}\left[\log\left|1 + (\frac{v_{i+1}}{v_0^-})^2\right| - \log\left|1 + (\frac{v_i}{v_0^-})^2\right|\right] \tag{3''}$$

knowing Δx_i and v_i , one deduces v_{i+1} from (3") then, knowing t_i one deduces t_{i+1} from (2")

* if A>0 : $$t_{i+1} - t_i = \frac{1}{2\sqrt{AB}}\left[\log\left|\frac{\sqrt{\frac{A}{B}} + v_{i+1}}{\sqrt{\frac{A}{B}} - v_{i+1}}\right| - \log\left|\frac{\sqrt{\frac{A}{B}} + v_i}{\sqrt{\frac{A}{B}} - v_i}\right|\right] = \frac{1}{2\sqrt{AB}}\left[\log\left|\frac{1 + \sqrt{\frac{B}{A}}v_{i+1}}{1 - \sqrt{\frac{B}{A}}v_{i+1}}\right| - \log\left|\frac{1 + \sqrt{\frac{B}{A}}v_i}{1 - \sqrt{\frac{B}{A}}v_i}\right|\right] \tag{2'''}$$

$$x_{i+1} - x_i = \Delta x_i = \frac{-1}{2 \cdot B}\left[\log\left|\frac{A}{B} - v_{i+1}^2\right| - \log\left|\frac{A}{B} - v_i^2\right|\right] = \frac{-1}{2B}\left[\log\left|1 - \frac{B}{A}v_{i+1}^2\right| - \log\left|1 - \frac{B}{A}v_i^2\right|\right] \tag{3'''}$$

Remark : the energy consumption per unit distance $\Delta E_i/\Delta x_i$ (engine on -> *ign* = 1 and engine off -> *ign* = 0) is a function of engine speed N_m , throttle opening α_{gaz} , gear ratio and wheel radius.

Remark : letting $\frac{A}{B} = v_0^2$ (constant), equations (2") et (3") become :

$$t_{i+1} - t_i = \Delta t = \frac{1}{2 \cdot B \cdot v_0}\left[\log\left|\frac{1 + \frac{v_{i+1}}{v_0}}{1 - \frac{v_{i+1}}{v_0}}\right| - \log\left|\frac{1 + \frac{v_i}{v_0}}{1 - \frac{v_i}{v_0}}\right|\right] \tag{2''''}$$

$$x_{i+1} - x_i = \Delta x = \frac{-1}{2 \cdot B}\left[\log\left|1 - (\frac{v_{i+1}}{v_0})^2\right| - \log\left|1 - (\frac{v_i}{v_0})^2\right|\right] \tag{3''''}$$

average speed on the segment x_i to x_{i+1} is

$$v_{ave} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} = -v_0 \cdot \frac{\left[\log\left|1 - (\frac{v_{i+1}}{v_0})^2\right| - \log\left|1 - (\frac{v_i}{v_0})^2\right|\right]}{\left[\log\left|\frac{1 + \frac{v_{i+1}}{v_0}}{1 - \frac{v_{i+1}}{v_0}}\right| - \log\left|\frac{1 + \frac{v_i}{v_0}}{1 - \frac{v_i}{v_0}}\right|\right]} \tag{4}$$

use of the *elementary driving model* Excel sheet

to simplify the development of the model, we have made some assumptions :
- the car runs on a **straight level road without wind**
- the mechanical friction factor *a* is a constant
- the aero dynamical friction force depends on the square of the running speed
- its factor depends only on the frontal area (effect of wet area is included in the aero dynamical friction factor/drag factor)
- the engine/motor torque is constant during motoring
- the gear ratio is fixed during the run (no clutch or tire slide)
- on each line (from line 14), minimum and maximum speeds are imposed with increasing difference : the start speed and end speed of a sequence are the same and equal to minimum speed, after start the engine is cut-off when the maximum speed is reached
- the engine/motor efficiency is constant, as well as the transmission efficiency
- the free wheel friction is included, taking into account its relative operating time, in the mechanical friction factor
- usually, there is a constant energy cost at each engine/motor start

column contents (from line 14) :

column A : minimum speed (start and end speed)
column B : maximum speed (speed at cut-off)
column C : distance run during the motoring phase
column D : duration of this phase
column E : distance run during the coasting phase
column F : duration of this phase
column G : total distance run during the sequence (motoring phase + coasting phase)
column H : duration of these 2 phases (the sequence)
column I : energy consumed by the engine/motor during motoring phase
column J : specific energy related to the total distance
column K : specific distance run per unit of energy (1kWh)
column L : average speed on the distance
column M : energy consumed (in kJ) for the length of the run
column N : minimum engine/motor speed corresponding to the minimum car speed (useful in the case of a speed limit imposed by a centrifugal clutch, for example)
column O : maximum engine/speed corresponding to the maximum car speed (useful in the case of a speed limit imposed by a mechanical constraint)
column P : specific distance run per liter of unleaded gasoline (or as an equivalent for other sources of energy)
column Q : approximate number of sequences during a lap
column R : motoring time ratio (depends mainly on propulsion force)